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## FINAL REPORT

on

# MODULAR SYSTEMS THEORY AND DESIGN ANALYSIS

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## TABLE OF CONTENTS

1.	Introduction . . . . .	1
2.	Main Results . . . . .	3
	2.1 Deciding on a Modular Design Approach . . . . .	3
	2.2 Modular Structuring of Connected Physical Systems . . . . .	5
	2.3 Macromodeling, Decoupling and Optimization . . . . .	6
	2.4 Economic and Reliability Considerations in Modular Design . . . . .	7
	2.5 Mathematical Theory of Modular Systems . . . . .	8
3.0	Conclusions . . . . .	9
4.	References . . . . .	10

## 1. Introduction

So far the modular system concept is centered around component standardization. In an attempt to reduce weapon systems' costs certain components have been declared as "modular" or better yet as standard. This type of declaration is commendable. A manager who makes such a declaration can be in a precarious position if he has limited information on the ramifications of "pouring such components in concrete". If a declared standard component causes future systems to have significantly poorer performance or to be much more complex and costly (i.e., have less overall modularity), the declared standards must be revised. Such standards will probably have a very short life-time when applied to elements of ultra-high performance systems unless the standards are based on "top-down" designs of all applicable systems. Thus, requirements and technology must be accurately forecast, all potentially high cost systems must be defined, all possible common modules must be cataloged, and the impact of overall system cost of declaring certain components as common modules must be assessed.

Modular systems can mean more than systems of standard components. Highly significant system life-cycle-cost savings can be obtained via the use of other attributes of modules. For example, we have stated in earlier reports that modules should be as energy/information/structurally independent of other modules as possible. This has been shown to offer advantages in system reliability and maintainability. This idea of independence and decoupling in design is slightly more complex (requires more decision making information) but it is being used in new weapon systems today. The so called line replacement units (LRU's) are no longer just replaceable at unit maintenance (for example, AUVM) but can be replaced without disturbing other modules and, in many cases, by a single tool.

In essence, systems should consist of a support structure (which can be shared by modules up to the point where the structure becomes a mechanism when modules are removed) and the modules themselves.

We have not developed a unified theory for the design and analysis of the ultimate in modular systems. We have developed basic concepts of "what modules are" and have used in design and analysis some powerful mathematics such as "dynamic programming", "tearing methods for decoupling", "modern algebra for modular design of dynamic system", and other "modern optimization methods". Roughly, our concept of the ultimate in modular systems is: the systems should perform their functions; the total life-cycle-cost of each system should be a minimum with maximum utilization of common elements; maintenance times should be minimum at all levels; and, reliability should be constrained to at least state-of-the-art levels.

Not all conceptual systems are optimum when they are modular. Cost, performance and the number of units, and the number of possible modular subsystems can limit the level of modularity to a trivial level; and, the best system could be a unitized system with even very limited use of standard hardware components. Our research into

human decision making in design has used the promising approach of treating uncertain qualitative factors via fuzzy semantic variables. The goal of this work has been to develop decision making computer programs for deciding whether or not to seek a modular design approach.

The starting point for a top-down design of a modular system is the identification of modular functions. In some systems separate functions can share a common module and the system can still maintain its simplicity and ultimate modular form. For example, in pointing and tracking systems for beamrider or laser designation a common optics module can be shared by two or more functional subsystems. Thus, one first identifies the independent functions and then decides via optimization methods how to modularize the functional subsystems for an optimal system. One approach is to design for the functional subsystems to share the maximum number of modules. With constraints only on the functions this can lead to a suboptimum (less than ultimate) modular system. Such decisions must be based on precise mathematics for partitioning systems with respect to: 1) structures, 2) reliability, 3) maintainability (and availability), 4) inventory, and 5) reduced dynamic sensitivity as well as functional optimization.

Design from the bottom-up does not appear to provide both a well functioning system and an economical system when the system is one requiring interdisciplinary design. However, many subsystems and some simple large scale systems like heat exchangers [1] can be ultimately modular when designed from the bottom-up. The analysis process for modular system design is of necessity different from analysis in a conventional design because, among other things, a continuum of design solution is generally not possible. Dynamic programming proved useful in handling bottom-up designs of the inherently discrete modular systems.

In addition to the research in modular systems design decision making we have:

1. applied a least squares method to the allocation of system resources (e.g. mass and power flow) among functional modules.
2. developed new methods of macromodeling for simulation and design of modular systems.
3. developed a new least squares algorithm for solving the partitioned (and sparse) nonlinear equations of the macromodels.
4. developed optimal group replacement times for deteriorated modules as observed during periodic review.
5. developed optimal processing costs and deferral cost of machining of scheduled modules with particular attention paid to the time value of money.
6. related reliability and equipment life to decreasing mean-residual-life functions and the concepts of negative memory, positive memory and no memory.
7. developed the algebraic relationships between decoupling indices of dynamical systems and the reachability or functional (Kronecker) indices.

## 2. Main Results

Modular systems as systems with complex functions are designed by interdisciplinary groups of engineers and scientists. Systems engineers who understand all design goals and functional interactions of components must play a key role in decisions regarding design. Thus, if work on "modular systems theory and design analysis" is to be fruitful it must produce new information, which is useful to these design people, and provide decision making methods. Our work has produced new information for deciding on a modular design. Concepts for defining a modular system and a module have been developed. New methods for determining reliability and maintenance strategies of modular systems were also developed. These results together with brief descriptions of the macromodeling work and algebraic systems theory research will now be described. (The Appendix contains published papers and reports).

### 2.1 Deciding on a Modular Design Approach

Commonly the designer must make decisions regarding both whether or not to seek a modular approach at all and, if so, which of several alternatives is most desirable. In problems of practical interest, these decisions are made difficult by the presence of multiple performance criteria which the design seeks to achieve, multiple constraints on the nature of the design, multiple prescribed features of each design, and economic considerations imposed by other systems in the class. A further complication is the high level of uncertainty regarding many significant factors and their interactions. This plus the noncommensurability of their quantitative estimation makes it extremely difficult for judgement factors to be satisfactorily included in the considerations.

Recent developments in the treatment of highly uncertain qualitative factors, using fuzzy semantic variables seem to offer a promising approach to this latter difficulty. In addition, research into human decision-making (Dawes [2]) suggests that even simple linear computer models can do much better than humans in making multiple criteria decisions. These factors suggest that a linear interaction model using fuzzy semantic variables would be clearly superior to a "seat-of-the-pants" decision approach by the designer. Effort has been directed toward the development of such a model.

The Design Selection Model involves the following major steps:

- 1) Select a set of performance criteria and requirements for the design. These might involve factors such as geometrical size, weight, spacing, chemical compatibility, temperature and humidity specification, maintenance requirements, yield, range, deployment flexibility, shelf life, electronic compatibility with other components, etc. This set of requirements and criteria should consist of distinct factors and be complete in the sense that no significant criteria are omitted.
- 2) The magnitude and time-dependency of direct influences of various proposed design approaches on the level of performance

of each of these criteria is then estimated and expressed in terms of fuzzy semantic variables. For example, magnitudes might be expressed as positive or negative enormous, large, substantial or minor effects. The time effects might be expressed as instantaneous, short or long term.

- 3) The interactions anticipated between levels of achievement of various goals are then similarly estimated. These interactions should result from influence means other than directly from the design choice itself.
- 4) An aggregation scheme appropriate to the design task is then chosen such that all of the levels of performance are suitably weighted for their significance in the particular situation and then combined into a single time-dependence performance index.
- 5) Appropriate numerical "meanings" are chosen for the various semantic terms used. These might, for example, be enormous =100, large =10, substantial =1, minor, = 0.1 etc.
- 6) The model is then programmed for a computer and stepped through the expected life span of the design to obtain a predicted performance-time history of each design option.
- 7) The numerical results are levels of meaning in semantic terms and are converted back into the semantic terms familiar to the designer. These semantic terms are then presented to the designer or other decision maker.
- 8) Further aggregation over time using again an appropriate criteria (such as the area under the performance/time curve, for example) can then be carried out to give a single overall index of performance for each design approach if desired.

Preliminary programming of this general approach has been carried out in APL. Hypothetical test data has yielded stable results through time which seem at least to be reasonable. Testing of such models is very difficult, however, since research on complex systems; by for example, Forrester [3], indicates that they frequently exhibit counter-intuitive behavior. Thus it is worse than meaningless to ask that the model behave in a fashion reasonable to human judgement not based on in-depth study.

Although no estimates can be made of the absolute accuracy of the predictions of this model approach, it appears quite certain from the research results quoted that models such as this one can make considerably better predictions in complex multiple criteria situations than can humans. Thus it appears highly desirable to develop at least simple performance models such as the one described above to assist designers in choosing among alternative modular design approaches.



## 2.2 Modular Structuring of Connected Physical Systems

In the design of complex physical systems the designer envisions parts which will go into making up the desired systems. The systems considered for modular design are usually a class of functionally similar systems, e.g. motorized land vehicles or modular air-to-surface weapons, since a modular class has economic advantages over a single modular design. The envisioned parts for the systems are mostly existing in other systems, do not fit together very well in the envisioned systems, and many are thought to be better produced in-house. The designer is lead by economic considerations to a top-down modular design, especially if he is to design a significant number of the systems in the class or is to design for in-house production one or more of the modules common to various members of the class. With a view of all systems in the class, one can reduce the complexity of the systems as well as that of new modules. (New modules are those at the forefront of the state-of-the-art and must be developed for the new system to perform its task.) Complexity is one of those things that modularity is not. Complexity can be a functional complexity in a dynamical system function sense (see Rosenbrock and Pugh [4]) but it must also be the number of maintenance steps (human operations), the number and degree of interconnections of modules, etc. Thus, in the physical modeling for function or in the economic modeling for maintainability, reliability, etc. we must express our models in terms of functional physical parameters of the system and physical parameters which reflect cost and complexity. Then, if we set design goals in terms of functions which depend on these parameters, we can design the physical structure by an optimal parameter allocation procedure.

We have chosen the following form for the design optimization problem. If we are given the independent variable  $x$  which gives the points in time or space or the number in a sequence of observations, etc.; and, if we determine desirable performance functions  $f_d(x)$  for the class of systems; and, if we can model the system performance in such a way that the output of the model is a function of system parameters,  $p$ , and expresses system performance corresponding to  $f_d(x)$ , then the output of the model,  $f(x,p)$ , is to be as "close" as possible to  $f_d(x)$  when the parameters,  $p$ , are optimally determined. This problem is usually formulated as a minimum norm problem on the space of functions  $f(x,p)$  or on the parameter space  $p$ . If the model is known to within the parameters,  $p$ , the latter space is used. Otherwise, one has the more general functional optimization and synthesis problems.

We have assumed that the model structure is well known but the arrangement of the system as described by the parameters are unknown. The norm chosen was the least squares function of  $p$ . The optimization criterion can be the sum of several least squares functions. In some system design problems it is possible to partition the set of parameters such that some parameters do not occur in all functions. Then the design problem can be solved by two or more optimization problems. For example, suppose  $f_d(t)$  is a desire per-

formance function of time,  $f_{d2}(x_h)$  is a desired inertia function in body fixed coordinates, and  $f_{d3}(x)$  is a desired maintenance function of component stacking order or order of interconnection. If the desired performance is influenced by only the parameter set  $p_1$ , the inertia is affected only by the set  $p_2$ , and the maintenance is affected by  $p_2$  and the set  $p_3$ , then we can formulate the optimal allocation problems. Find the parameters in the sets  $p_1$ ,  $p_2$ , and  $p_3$  such that

$$J_1 = \sum_{s=1}^S \int_0^T \left\{ r_{d1}^s(t) - r_1^s(t, p_1) \right\}^2 dt$$

$$J_2 = \sum_{s=1}^S \int_{V(p_2)} \left\{ r_{d1}^s(t) - r_1^s(t, p_1) \right\}^2 dt$$

$$+ \sum_{s=1}^S \sum_{k=1}^K \left\{ r_{d3}^s(x_k) - r_3^s(y_k, p_2, p_3) \right\}^2$$

are minimum. The optimization is done over the entire class of system,  $s=1, \dots, S$ . Minimizing  $J_1$  is independent of minimizing  $J_2$ .

This type of optimal allocation problem cannot be formulated in great detail in terms of the fundamental physical phenomena, since the problem would become too complex and require enormous amounts of computing memory and time. Thus, one is lead to formulating the functions  $r_1^s$  in terms of only important details of the desired behavior as a function of physical parameters.

Only simple example problems have been formulated and solved so far. This research has pointed out the need for macromodels of both physical system performance and economic performance in terms of fundamental physical design parameters.

### 2.3 Macromodeling, Decoupling and Optimization

The ever increasing size of circuits and systems that engineers are designing has led to the development of many special purpose simulation techniques, e.g., macromodeling. The task of trying to decide the relative merits of various simulations techniques is made difficult by the lack of a unifying framework in which to study the relationship between modeling for simulation and the actual simulation procedures. The aim of our work has been threefold: To review existing large scale simulation procedures and develop a unifying structure for analyzing simulation procedures and to formulate some potentially useful new large scale simulation procedures. This work is summarized in the paper "Simulation Procedures for Large Scale Electronic Systems."

In addition to the above we have developed a new algorithm for solving sparse  $n \times n$  sets of nonlinear algebraic equations. This

algorithm is like the Levenberg-Marquardt algorithm in that at each iteration the step size taken affects the direction selected to search, this direction lying somewhere between the Newton and gradient directions. Unlike the Levenberg-Marquardt schemes the sparsity of the original equations is preserved and can be exploited. This work is summarized in the paper "A Modified Least Squares Algorithm for Solving Sparse  $N \times N$  Sets of Nonlinear Equations." (See Appendix).

#### 2.4 Economic and Reliability Considerations in Modular Design

Research into several classical areas of design economics for new modular systems was performed. One area was on the problem of scheduling of repair of failed modular units considered as aggregated components in multicomponent systems. Another area of research was the description of a replacement policy for a modular unit in which the components are subject to strict deterioration but no failure. A third area of research was on the development of mean residual life functions for modular systems. In all instances appropriate mathematical models were developed and explicit solutions were obtained for several cases.

Scheduling repair (or replacement) of failed modular units in multicomponent systems was investigated as follows. By viewing each failed module as a job to be repaired by a processor and the processor as a single machine, optimal permutation type schedules were obtained for a class of  $n$  jobs one machine type problems using as criteria i) the total processing cost, ii) the total deferral cost and iii) the total processing and deferral costs. Seven models were developed including cases when the processor deteriorates with usage, and when resetting of the processor is possible at the completion of each job. The results can be applied to determine an optimal sequencing scheme to repair an arbitrary number of failed modules in a complex system in which a single repair facility is available. A report of this work can be found in the Appendix under the title of "Permutation Type Schedules on a Single Machine under Cost Criteria."

The modular replacement problem with units subject to strict deterioration was considered as follows. For an  $n$ -component modular unit in which each component is subject to strict deterioration, a periodic group replacement policy (module replacement) was assumed when the level of deterioration of all  $n$  components reaches a given vector value. The underlying multidimensional renewal equation was derived as well as expressions for the mean time between replacements. The statistical characteristics of component deterioration can thus be related to module replacement. This work is presented in the Appendix in the paper entitled "A Group Replacement Problem under Deteriorating Conditions".

A solution to the mean residual life problem for modular units was obtained in terms of properties of negative, positive and no memory and these were related to equipment life and reliability. The paper, "Families of Distributions with Positive Memory Derived from the Mean Residual Life Function", summarizes this research.

## 2.5 Mathematical Theory of Modular Systems

The following topic was the center of interest: Relationship between the reachability (KRONECKER) indices and the decoupling indices. According to ROSENBROCK's theorem concerning the effect of feedback on system dynamics (see State-space and Multivariable Theory, Wiley, 1970, Theorem 4.2, p. 190), the degrees  $\deg \phi_i$  of the invariant factors  $\phi_i$  of any (closed-loop) system matrix  $F_*$  must satisfy the inequalities

$$\deg \phi_1 \geq K_1, \quad (*)$$

$$\deg \phi_1 + \deg \phi_2 \geq K_1 + K_2,$$

etc., where  $K_1 \geq K_2 \geq \dots \geq 0$  are the KRONECKER indices of the fixed pair  $(F, G)$  describing the open-loop input-state behavior of the system to be controlled. (See KALMAN, "Kronecker Invariants and feedback", Proc. NRL Conference on Differential Equations, 1971; Academic Press, 1972.) Since the systems in question are assumed reachable, it is always assumed that

$$\sum \deg \phi_i = n \quad \text{and} \quad \sum K_i = n$$

Moreover,

$$\deg \phi_i \geq 0 \quad \text{and} \quad K_i \geq 0 \quad \text{for all } i.$$

These two properties show that (\*) can be interpreted as a partial order between partitions. In other words, ROSENBROCK's theorem gives an inequality, in terms of this partial order, between the degrees of invariant factors and the KRONECKER indices. Similar inequalities arise in connection with the study of the feedback indices. In short, the partial order (\*) plays a basic role in the study of structural indices in problems of feedback, decoupling, and modularity.

Using these ideas, new results have been obtained concerning the possibility of system modification by means of feedforward signals. These results complement well-known facts about the possibility of system modification by means of feedback. For example, we can reinterpret ROSENBROCK's theorem in two different ways:

(i) The generic KRONECKER indices (all  $\phi_i$  approximately equal) are at the bottom of the partial order (\*). Therefore ROSENBROCK's theorem is automatically true. (No proof needed, except the minimality of the generic indices with respect to the partial order.)

(ii) Given any system  $(F, G)$  with prescribed invariant factors for  $F$ , we can obtain (by feedforward modification of  $G$ ) any KRONECKER index below  $\deg \phi$  in the partial order.

Further elaboration of these ideas should provide a much simpler picture of questions of decomposition, modularity etc.; heavy algebraic manipulations are avoided by concentrating on properties of the indices.

Vigorous pursuit of this line of research is required if complex modular systems are to be developed from reasonably simple analytical methods.

### 3.0 Conclusions

One could define modules and modular systems in a number of different ways such that the definition is successful in defining a worthwhile and useful design. Presently, these definitions would be quite narrow. For example, one could say that a module is a part of a system such that the module is minimally connected to the other parts of the system and define "minimally" by the degree of decoupling observed in the modeling equations. This would not lead to the ultimate modular system configuration broadly defined in the Introduction. Thus, one must do both performance modeling and economic systems modeling in terms of common system design parameters. The use of "overlays" of the various modeling results described in the parameter space has been considered a viable aid to ultimate design.

Existing systems which seem to have various degrees of "modularity" have been analyzed to determine the basic attributes of modules. These attributes, which have been discussed throughout this report and in previous reports, are more than standardization and include advantages in the design process as well as savings in time and money for maintenance. Again, economic considerations are of primary importance. Economy in design, economy of ownership, etc. must influence a specific design. Thus, these must be related to the physical parameters of a design or class of designs.

Computer aids have been developed for economy in the design process as well as for decoupling the governing equations for various designs. Such computer aids are necessary in complex system designs and the process of modular design must be many faceted if the resulting class of modular systems is to be ultimate. The system itself need not be complex and the degrees of complexity of the models can be low. For example, if we think in terms of a dynamical systems model in terms of linear differential equations, then the degree of complexity is related to the degree of the characteristic polynomial [4]. The degree of complexity will increase with the degrees of complexity of (1) disturbances to the system which the system must overcome and (2) the prescribed performance which the system must follow. Thus, the model should contain only essential (sensitive) descriptions for disturbance rejection and following of prescribed performance.

In the original proposal for a five year program it was proposed to investigate modular systems for natural decoupling of state space models corresponding to the physical modular boundaries. It is concluded that any natural state variable decoupling caused by the intrinsic properties of modules of ultimately modular systems are insignificant. Coupling and decoupling of modules are better described in a physical parameter space. Reasonably uncomplex mathematical methods must be developed for system decomposition in parameter space.

The stage is set for new research in the economic design of modular systems wherein reliability, maintainability and availability (RAM) are

considered in terms of physical design parameters. This will require that the system engineers whose primary responsibility is RAM interfaces more with the engineers responsible for subsystems.

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